The Language of Statistical Decision Making

Lecture 3
Section 1.3

Robb T. Koether

Hampden-Sydney College

Mon, Jan 23, 2012

Outline

Review Quiz

Example (Review Quiz)

1. The null hypothesis
   (a) Is probably true.
   (b) Gets the benefit of the doubt.
   (c) Can be proven true.
   (d) Requires proof.

Review Quiz

Example (Review Quiz)

2. If the population is a set of registered voters, then the sample is
   (a) The percentage that vote Republican.
   (b) Those voters who actually voted.
   (c) A set of registered voters.
   (d) Their party affiliation.

Review Quiz

Example (Review Quiz)

3. A researcher is studying unemployment. Which of the following could be his null hypothesis?
   (a) The unemployment rate has increased.
   (b) The unemployment rate has decreased.
   (c) The unemployment rate has stayed the same.
   (d) Any one of the above.

Review Quiz Answers

Example (Review Quiz Answers)

1. (b) Gets the benefit of the doubt.
2. (c) A set of registered voters.
3. (c) The unemployment rate has stayed the same.

Hypotheses


IT SEEMS THERE IS A DIFFERENCE BETWEEN A "HYPOTHESIS" AND A "GUESSTIMATE."

Outline

Errors

- In the coin-tossing experiment, the hypotheses were
  - H_0 : The coin is fair.
  - H_1 : The coin is not fair.
- Based on the evidence, the null hypothesis was accepted.

Errors

- Do we now *know* that the coin is fair?
- Or could we be mistaken?
- Why?

Errors

- Had we gotten 10 tails instead of 7, we would have concluded that the coin was not fair.
- Would we then have *known* that the coin was not fair?
- Or could we have been mistaken?
- Why?

Two Types of Error

Definition (Type I Error)

A Type I error is to reject the null hypothesis when it is true, i.e., when it should have been accepted.

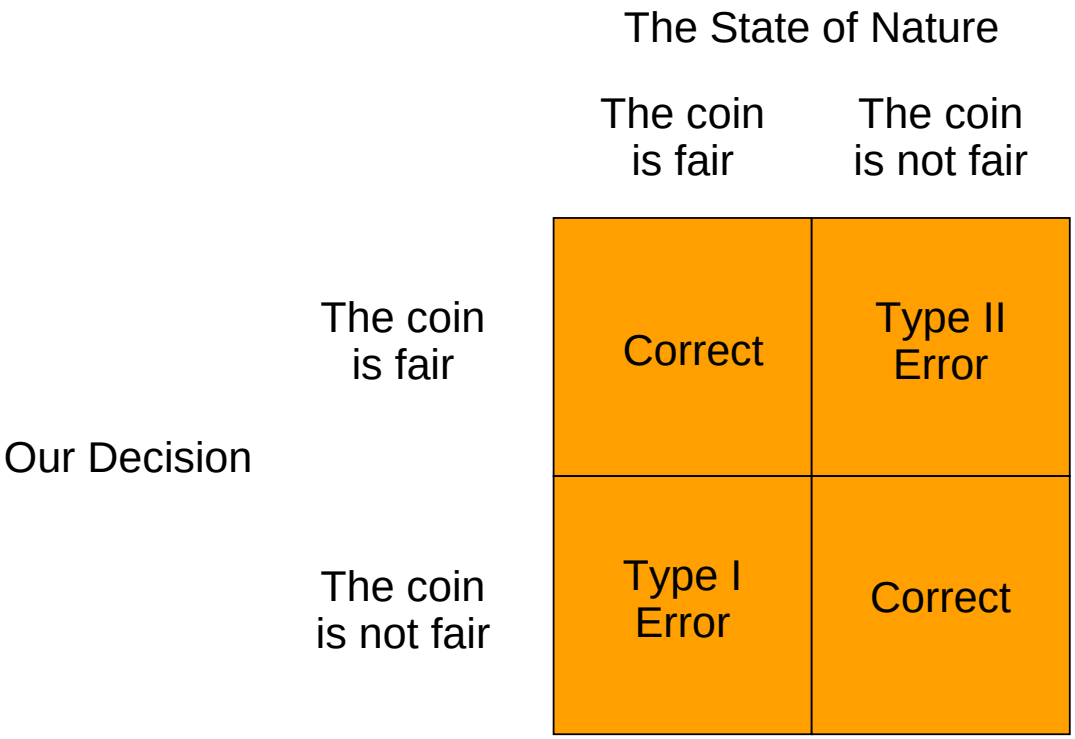
Definition (Type II Error)

A Type II error is to accept the null hypothesis when it is false, i.e., when it should have been rejected.

Decisions and Errors

Our Decision	The State of Nature: H_0 is true	The State of Nature: H_0 is false
Accept H_0	Correct	Type II Error
Reject H_0	Type I Error	Correct

Decisions and Errors



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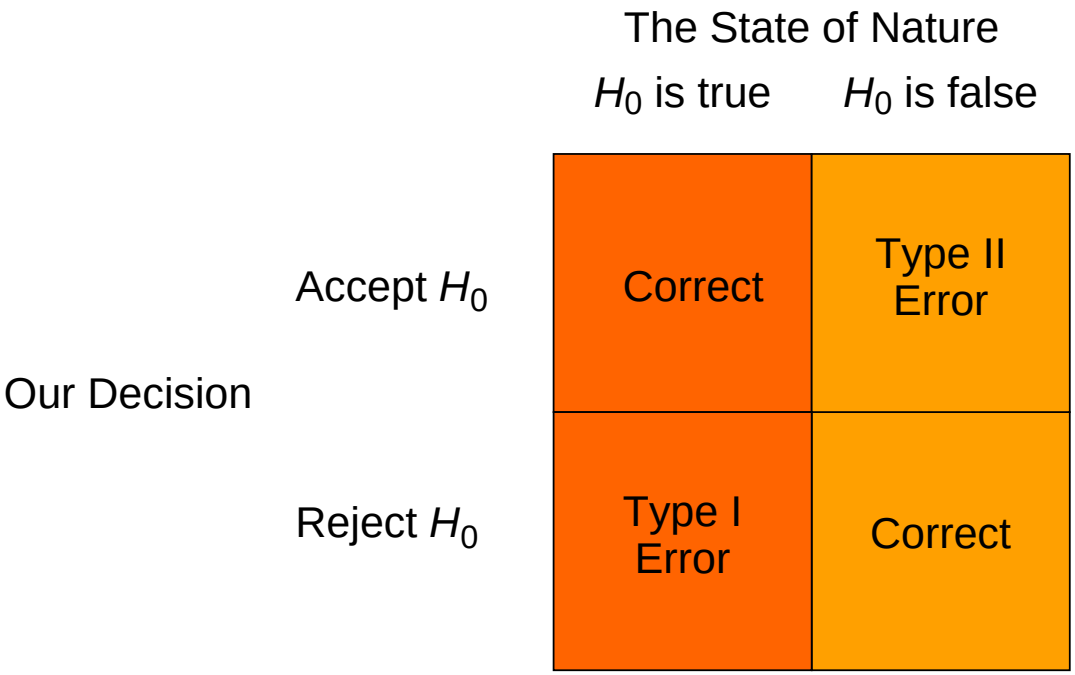

Types of Error

Example (Types of error)

- A woman administers a home pregnancy test on herself.

Types of Error

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- A woman administers a home pregnancy test on herself.
- She may or may not be pregnant.

Types of Error

Example (Types of error)

- A woman administers a home pregnancy test on herself.
- She may or may not be pregnant.
- What should be the null hypothesis?

Types of Error

Example (Types of error)

- The hypotheses should be
  - H_0 : She is not pregnant.
  - H_1 : She is pregnant.

Types of Error

Example (Types of error)

- There are two possible diagnoses (conclusions):

Types of Error

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  - The test is positive, i.e., the woman is pregnant.
  - The test is negative, i.e., the woman is not pregnant.

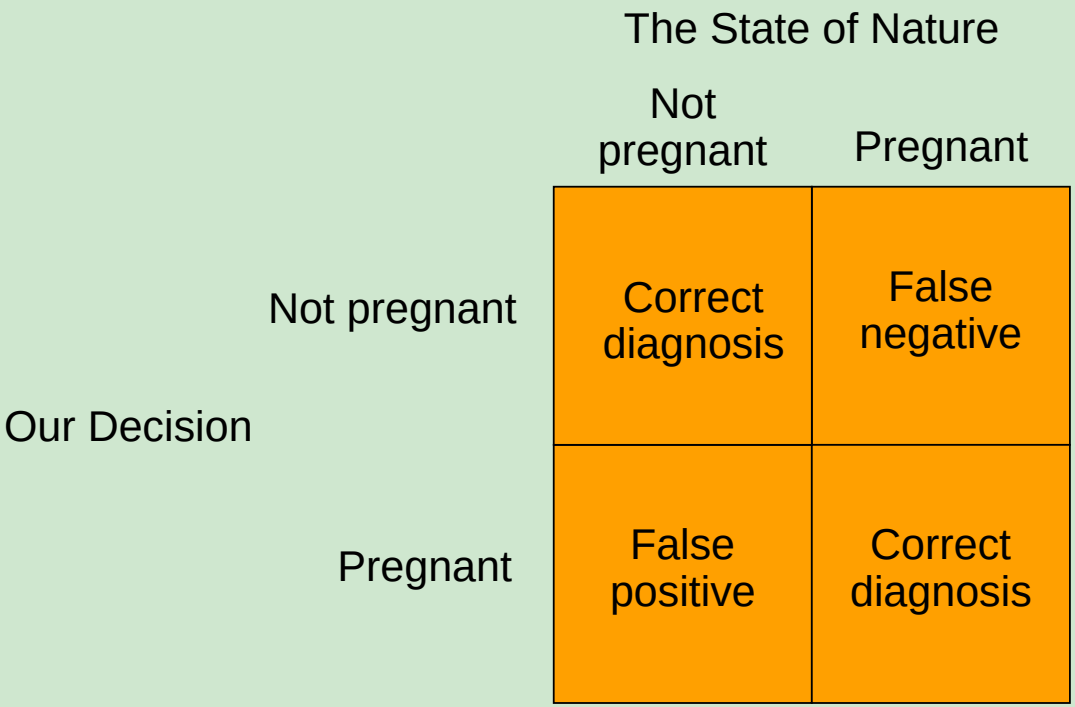
Types of Error

Example (Types of error)

- There are two possible diagnoses (conclusions):
  - The test is positive, i.e., the woman is pregnant.
  - The test is negative, i.e., the woman is not pregnant.
- Describe a Type I error.
- Describe a Type II error.

Types of Error

Example (Types of error)



Outline

Statistical Significance

Definition

The data are called statistically significant if their deviation from what would be expected under the null hypothesis is too great to be attributed to sampling error (chance).

Statistical Significance

- In the coin-tossing experiment, the results did not deviate *much* from what we would expect of a fair coin.
- Therefore, the results were *not* statistically significant.
- Therefore, therefore we did *not* reject the null hypothesis.

Significance Level

Definition (Significance Level)

The significance level of a test is the likelihood of rejecting H_0 when it is true, i.e., the likelihood of committing a Type I error, based on the design of the experiment.

- The symbol α represents the likelihood of a Type I error.
- The symbol β represents the likelihood of a Type II error.

Significance Level

Example (Significance level)

- Suppose that we have two unusual dice.
  - Die A rolls a 1 80% of the time and a 4 only 20% of the time.
  - Die B rolls a 1 only 10% of the time and a 4 90% of the time.

Outcome	Die A	Die B
1	0.80	0.10
4	0.20	0.90

- Visually, the two dice are indistinguishable.

Significance Level

Example (Significance level)

- We pick up one of the dice.
- Let the hypotheses be
  - H_0: We picked up die A.
  - H_1: We picked up die B.
- We will roll the die one time and, based on the outcome, decide which die we think it is.

Significance Level

Example (Significance level)

- What should be our decision rule for choosing between the two hypotheses?
- That is, if the die turns up 1, which hypothesis do we choose?

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- What if it turns up 4?
- Describe a Type I error.

Significance Level

Example (Significance level)

- What should be our decision rule for choosing between the two hypotheses?
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- What if it turns up 4?
- Describe a Type I error.
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Example (Significance level)

- What should be our decision rule for choosing between the two hypotheses?
- That is, if the die turns up 1, which hypothesis do we choose?
- What if it turns up 4?
- Describe a Type I error.
- Describe a Type II error.
- What is the value of α?

Significance Level

Example (Significance level)

- What should be our decision rule for choosing between the two hypotheses?
- That is, if the die turns up 1, which hypothesis do we choose?
- What if it turns up 4?
- Describe a Type I error.
- Describe a Type II error.
- What is the value of α?
- What is the value of β?

Outline

Increased Sample Size

- It seems that more evidence should lead to a more reliable conclusion.
- Can we reduce the probability of error by getting a larger sample?

Increased Sample Size

Example (Increased sample size)

- Suppose now that we roll the chosen die three times and use the *average* of the three rolls to make our decision.
- We must get either
  - Three 1s, with an average of 1.
  - Two 1s and one 4, with an average of 2.
  - One 1 and two 4s, with an average of 3.
  - Three 4s, with an average of 4.

Increased Sample Size

Example (Increased sample size)

- The following table summarizes the situation.

Outcome	Average	Die A	Die B
Three 1s	1	0.512	0.001
Two 1s, one 4	2	0.384	0.027
One 1, two 4s	3	0.096	0.243
Three 4s	4	0.008	0.729

Increased Sample Size

Example (Increased sample size)

- What would be a good decision rule for deciding which die it is?
- Based on this decision rule,
  - What is α?
  - What is β?

Outline

Assignment

Homework

- Read Section 1.3.3, pages 11 - 15.
- Let's Do It! 1.5, 1.6.
- Page 67, exercises 5 - 9, 11 - 12 (I omitted #10).